White Paper

Calculate Surface Areas and Cross-sectional Areas in Vessels with Dished Heads

Executive Summary

Vessels with dished heads are used throughout the chemical process industries for storage, phase separation and chemical conversion. Consequently, rigorous treatment of the volume, surface area and cross-section area of such vessels for varying liquid depth is required for many applications. One such application is process simulation which is used pervasively throughout the process industries for offline process design and analysis and within many plant operations applications including model predictive control, fault detection and diagnosis, and operator training. When simulating the dynamics of a process, the size, shape and orientation of vessels has a strong influence on the trajectories predicted for the variables of the process. Rigorous treatment of the volume, surface area and cross-sectional area of vessels with dished heads is particularly important when simulating critical process safety operations, such as emergency depressuring, also widely known as blowdown. Low equipment wall temperatures encountered during depressuring can lead to brittle fracture of the metallic materials of which equipment is made. For new facilities, it is important to predict depressuring times and the temperatures accurately to allow equipment designers to select the most cost effective materials. For example, if stainless steel is selected where carbon steel would have been adequate, equipment costs could be twice as high or more. For existing facilities, reassessment of the temperature during depressuring can lead to changes in operating conditions or changes in process hardware.

Other authors have published exact formulas, together with their derivation, for the total volume of dished heads and for the volume corresponding to any liquid depth within vertically and horizontally oriented cylindrical vessels with dished heads. This paper extends this previous work to exact formulas for surface area and cross-sectional area corresponding to any liquid depth within vertically and horizontally oriented cylindrical vessels with dished heads, which are required, for example, for modeling of heat transfer between the fluid holdup in a vessel and its walls and between adjacent zones of the vessel during depressuring.

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Background

This article presents formulae for calculating the surface area below any liquid depth (the liquid surface area) and the cross-sectional area at any liquid depth (the cross-sectional area) within vertically and horizontally oriented cylindrical vessels with dished heads. The formulae apply for all dished heads characterized by two radii of curvature, that is, torispherical heads. Standards for torispherical heads have been defined by ASME (ASME Flanged and Dished (F&D), ASME 80/10 F&D, ASME 80/6 F&D) and by Deutsches Institut für Normung[1] (DIN 28011, DIN 28013). Other torispherical heads include the Standard F&D, Shallow F&D, and semielliptical heads, for which the shapes vary somewhat from one fabricator to another and hemispherical heads. Custom torispherical heads can also be fabricated.

Vessels with dished heads are used throughout the chemical process industries for storage, phase separation and chemical conversion. Consequently, rigorous treatment of the liquid volume, liquid surface area and cross-section area of such vessels is required for many applications. One such application is process simulation which is used pervasively throughout the process industries for offline process design and analysis and within many plant operations applications including model predictive control, fault detection and diagnosis, and operator training. When simulating the dynamics of a process, the size, shape and orientation of vessels can have a strong influence on the time based trajectories predicted for the variables of the process.

Rigorous treatment of the liquid volume, liquid surface area and cross-sectional area of vessels with dished heads is particularly important when simulating critical process safety operations, such as emergency de-pressuring, also widely known as blowdown. Low equipment wall temperatures encountered during de-pressuring can lead to brittle fracture of the metallic materials of which equipment is made. For new facilities, it is important to predict de-pressuring times and temperatures accurately to allow equipment designers to select the most cost effective materials that will withstand the demands of de-pressuring. For example, if stainless steel is selected where carbon steel would have been adequate, equipment costs could be twice as high or more. For existing facilities, reassessment of the temperature during de-pressuring can lead to changes in operating conditions or changes in process hardware to ensure safe operation during de-pressuring.

Crookston and Crookston [1] present exact formulae, together with their derivation, for the capacity of torispherical heads and for the liquid volume (the volume corresponding to any liquid depth) within vertically and horizontally oriented cylindrical vessels with torispherical heads. This article extends their work to exact formulae for liquid surface area and cross-sectional area. The formulae presented can be used with any engineering unit for length: meters, centimeters, feet, etc., provided it is used consistently through all the formulae. The engineering unit for area is the square of the engineering unit chosen for length.

Characterizing Torispherical Heads

Figure 1 shows that the cross-section of a torispherical head containing its central axis is comprised of three parts: a dish and a knuckle, each of which is characterized by a circular arc, and a flange. The head is formed simply by rotating the cross-section about the central axis. The knuckle smoothly joins the dish to the flange which smoothly joins the head to the cylinder of the vessel.
It is convenient to characterize torispherical heads by two dimensionless parameters: the dish radius factor and the knuckle radius factor. The radius factors are simply formed by dividing the corresponding inside radius by a characteristic length. The characteristic length can be either the inside diameter of the flange, that is, the inside diameter of the cylinder of the vessel, which yields

[1] German Institute for Standardization

Equation 1

$$f_d = R_d/D_i \geq 0.5$$

$$f_k = R_k/D_i \leq 0.5$$

or the outside diameter of the flange, that is, the outside diameter of the cylinder of the vessel. Table 1 shows dish and knuckle radius factors for several standard and fabricator-specific head styles. For some head styles the characteristic length is the inside diameter while for others the outside diameter is used. However, for the formulae presented herein, the characteristic length of the dish and knuckle radius factors must be the inside diameter. Hence, when the characteristic length associated with f_d^F and f_k^F provided in Table 1, or obtained from other sources, is the outside diameter, to use the formulae presented herein the dish and knuckle radius factors must be revised according to

Equation 2

$$f_d = f_d^F D_o/D_i \geq 0.5$$

$$f_k = f_k^F D_o/D_i \leq 0.5$$

where

Equation 3

$$D_i = D_o - 2t$$

By introducing these dimensionless parameters, the specification of most torispherical heads becomes independent of the physical size of the vessel. The Standard F&D and Shallow F&D head styles are exceptions. For these head styles, Table 1 shows that the knuckle radius is a fixed value or changes in a stepwise manner with the diameter of the vessel. For these head styles, the knuckle radius factor cannot be determined until the diameter of the vessel is specified.

Radius as a Function of Axial Position

In this section we summarize the relationships derived by Crookston and Crookston for the inside radius of the vessel at any position along its central axis. As for liquid volume, these relationships are a key aspect in the development of the formulae for liquid surface area and cross-sectional area. The relationships are presented in the context of a vertically oriented vessel. However, they apply equally to a horizontally oriented vessel simply by reading bottom as left, top as right, and height as length.

First, referring to the coordinate system shown in Figure 1, the vessel radius, x, and the axial position within the vessel, y, are transformed to dimensionless variables by defining.

Equation 4

$$\alpha = y/D_i$$

$$\beta = x/D_i$$

Figure 2 is a re-expression of Figure 1 in these dimensionless coordinates. It also shows the radius vectors for the vessel at $\alpha = \alpha_1$, the axial position where the dish joins the knuckle of the bottom head, and at $\alpha = \alpha_2$, and the axial position where the knuckle joins the flange of the bottom head.

The dish part of the bottom head, designated as Region 1 of the vessel, is a spherical cap. Using the coordinate system shown in Figure 2, the cross-section containing its central axis is an arc of the circle with radius f_d and centre $(0, f_d)$ so that for $0 \leq \alpha \leq \alpha_1$ the vessel radius, β, is related to the axial position, α, according to

Equation 5

$$\beta^2 + (\alpha - f_d)^2 = f_d^2$$

The axial position and vessel radius where the dish joins the knuckle of the bottom head are derived using basic trigonometric relationships for a right-angled triangle:

Equation 6

$$\alpha_1 = f_d\left(1 - \sqrt{1 - \left(\frac{1/2 - f_k}{f_d - f_k}\right)^2}\right)$$

Equation 7

$$\beta_1 = f_d\left(\frac{1/2 - f_k}{f_d - f_k}\right)$$

For a hemispherical head, $f_k = f_d = 0.5$ so that α_1 and β_1 are indeterminate. In fact, a hemispherical head is characterized by a single circular arc which can correspond to either the dish or the knuckle of the torispherical head. However, both conceptually and computationally, it is convenient for all the torispherical head styles to have both a dish and a knuckle. This is easily accomplished by choosing any value for α_1 in the range $0 < \alpha_1 < 0.5$, for example $\alpha_1 = 0.25$. The corresponding value for β_1 is found by substituting $\alpha = \alpha_1$ into Equation 5.

The knuckle part the bottom head, designated as Region 2 of the vessel, is a sector of a ring torus. Again, using the coordinate system shown in Figure 2, the cross-section through its central axis is an arc of the circle with radius f_k and with its centre at $(1/2 - f_k, \alpha_2)$ so that for $\alpha_1 \le \alpha \le \alpha_2$ the vessel radius is related to the axial position according to

Equation 8

$$\left(\beta - (1/2 - f_k)\right)^2 + (\alpha - \alpha_2)^2 = f_k^2$$

The axial position where the knuckle joins the flange of the bottom head is again derived using basic trigonometric relationships for a right-angled triangle:

Equation 9

$$\alpha_2 = f_d - \sqrt{f_d^2 - 2f_d f_k + f_k - 1/4}$$

Where the knuckle joins the flange of the bottom head, the radius of the head equals the radius of the vessel so that

Equation 10

$$\beta_2 = 1/2$$

The cylindrical part of the vessel, designated as Region 3 of the vessel, includes the cylinder of the vessel and the flanges of the heads. The radius for the cylindrical part of the vessel is constant and is simply half of the inside diameter. That is, for $\alpha_2 \le \alpha \le \alpha_3$

Equation 11

$$\beta_3 = 1/2$$

The axial position at the top of the cylindrical part of the vessel is

Equation 12

$$\alpha_3 = \alpha_2 + H_{tt}/D_i$$

or equivalently

Equation 13

$$\alpha_3 = H/D_i - \alpha_2$$

The tan-tan height of the vessel, H_{tt}, is the height of the cylinder of the vessel plus the heights of the flanges of the heads. Combing Equation 12 and Equation 13 yields

Equation 14

$$H = H_{tt} + 2\alpha_2 D_i$$

For the knuckle and dish of the top head, designated as Regions 4 and 5 of the vessel, due to the symmetry of the vessel, the relationship between the vessel radius and axial position can be readily obtained by replacing α with $(\alpha_5 - \alpha)$ in the relationships for the knuckle and dish of the bottom head, respectively. For Region 4, we have

Equation 15

$$\left(\beta - (1/2 - f_k)\right)^2 + (\alpha_5 - \alpha - \alpha_2)^2 = f_k^2$$

and

Equation 16

$$\alpha_4 = \alpha_3 + (\alpha_2 - \alpha_1)$$

For Region 5, we have

Equation 17

$$\beta^2 + (\alpha_5 - \alpha - f_d)^2 = f_d^2$$

and

Equation 18

$$\alpha_5 = \alpha_4 + \alpha_1 = \alpha_3 + \alpha_2 = H/D_i$$

Cross-sectional Area as a Function of Liquid Depth for Vertical Vessels

The cross-sectional area for a specified liquid depth within any vessel, regardless of its orientation, is the area of intersection of a horizontal plane at the liquid depth with the vessel. For a vertically oriented vessel, this intersection forms a circle so that the cross-sectional area is

Equation 19

$$A^V = \pi x^2 = \pi D_i^2 \beta^2$$

The cross-sectional area for a particular region of the vessel is found by simply solving the relationship relating its vessel radius to axial position for β and then substituting for β in Equation 19. This yields Equation 20 through Equation 24.

Equation 20

$$A_1^V = \pi D_i^2 (f_d^2 - (\alpha - f_d)^2)$$

Equation 21

$$A_2^V = \pi D_i^2 \left((1/2 - f_k) + \sqrt{f_k^2 - (\alpha - \alpha_2)^2} \right)^2$$

Equation 22

$$A_3^V = \pi D_i^2 / 4$$

Equation 23

$$A_4^V = \pi D_i^2 \left((1/2 - f_k) + \sqrt{f_k^2 - (\alpha_5 - \alpha - \alpha_2)^2} \right)^2$$

Equation 24

$$A_5^V = \pi D_i^2 (f_d^2 - (\alpha_5 - \alpha - f_d)^2)$$

Surface Area as a Function of Liquid Depth for Vertical Vessels

Region 1 – Dish of the Bottom Head

The intersection of a horizontal plane at any liquid depth within the dish of the bottom head forms the base of a "zone and segment of one base" of a sphere. Applying the mensuration formulae for a sphere [2], the liquid surface area of Region 1 is

Equation 25

$$s_1^V = 2\pi R_d y = 2\pi D_i^2 f_d \alpha$$

Region 2 – Knuckle of the Bottom Head

The surface area below any liquid depth within Region 2 is found by integrating the outer surface area of an infinitely thin horizontal slice, depicted in Figure 3, from the bottom of the knuckle to the liquid depth. However, we cannot use a slice with straight sides. Notice how much longer is $d\sigma$ than $d\alpha$ in Figure 3. Using a slice with straight sides introduces a significant error. Instead, we use a slice with a slanted side. In fact, the slice is a truncated cone. The mensuration formula for the surface area of an infinitesimally thin truncated cone with a dimensionless side length of $d\sigma$ and a dimensionless radius of β is

Equation 26

$$s = 2\pi D_i^2 \beta d\sigma$$

Since by the Pythagorean Theorem $d\sigma^2 = d\beta^2 + d\alpha^2$, we find that

Equation 27

$$s = 2\pi D_i^2 f_k \frac{\beta}{\sqrt{f_k^2 - (\alpha - \alpha_2)^2}} d\alpha$$

The liquid surface area is then obtained by substituting for β from Equation 8 and integrating from the bottom of the region:

Equation 28

$$s_2^V = 2\pi D_i^2 f_k \left(\alpha - \alpha_1 + (1/2 - f_k) \left(\sin^{-1}\left(\frac{\alpha - \alpha_2}{f_k}\right) - \sin^{-1}\left(\frac{\alpha_1 - \alpha_2}{f_k}\right) \right) \right)$$

Region 3 – Cylindrical Part of the Vessel

The liquid surface area for Region 3 is simply the surface area of a cylinder:

Equation 29

$$s_3^V = \pi D_i^2 (\alpha - \alpha_2)$$

Regions 4 and 5 – Top Head of the Vessel

When the liquid depth is in Region 4 or 5, the knuckle and dish of the top head, the liquid surface area is obtained by subtracting the surface area of the region above the liquid depth from the total surface area of the region. For Region 4 and 5, due to the symmetry of the vessel, the surface area above a vertical position α within the region equals the liquid surface area for vertical position $(\alpha_5 - \alpha)$ within Region 2 and 1, respectively, so that

Equation 30

$$s_4^V = S_4 - 2\pi D_i^2 f_k \left(\alpha_5 - \alpha - \alpha_1 + (1/2 - f_k) \left(\sin^{-1}\left(\frac{\alpha_5 - \alpha - \alpha_2}{f_k}\right) - \sin^{-1}\left(\frac{\alpha_1 - \alpha_2}{f_k}\right) \right) \right)$$

Equation 31

$$s_5^V = S_5 - 2\pi D_i^2 f_d (\alpha_5 - \alpha)$$

Surface Area of the Vessel

The total surface area of each of the five regions, $S_i, i = 1, \ldots, 5$, is found by substituting $\alpha = \alpha_1$ in Equation 25 and $\alpha = \alpha_i, i = 2, \ldots, 5$ in Equation 26+i. However, due to the symmetry of the vessel, $S_4 = S_2$ and $S_5 = S_1$ so that the total surface area of the vessel is

Equation 32

$$S = 2(S_1 + S_2) + S_3$$

When the liquid depth is in Region i, the liquid surface area of the vessel is the sum of the total surface areas for the regions below Region i plus the liquid surface in Region i, that is,

Equation 33

$$s^V = \sum_{j=1, i>1}^{i-1} S_j + s_i^V$$

Cross-Sectional Area as a Function of Liquid Depth for Horizontal Vessels

The liquid depth in a horizontally oriented vessel, d, is referenced to the bottom of the cylinder of the vessel and, hence, has values in the range $0 \le d \le D_i$. The corresponding dimensionless liquid depth is defined by

$$\delta = d / D_i$$

Region 3 – Cylindrical Part of the Vessel

The intersection of a horizontal plane with the cylindrical part of a horizontally oriented vessel forms a rectangle. The length of the rectangle in the axial direction is the tan-tan length of the vessel. Figure 4 shows that the other side of the rectangle is a chord of the circle which is the transverse cross-section of the cylindrical part of the vessel. Applying the mensuration formulae for a circle, the length of this chord is

Equation 34

$$c = 2\sqrt{d(D_i - d)} = 2D_i\sqrt{\delta(1-\delta)}$$

Hence, the cross-sectional area for Region 3 is

Equation 35

$$A_3^H = cH_{tt} = 2D_i^2\sqrt{\delta(1-\delta)}(\alpha_3 - \alpha_2)$$

Region 2 – Knuckle of the Left Head

The calculation of the cross-sectional area of the knuckle of the left head at any liquid depth within the vessel is very complex, involving the intersection of a horizontal plane with the portion of the ring torus of which the knuckle is a part. This ring torus is defined in a variant of our dimensionless Cartesian coordinates by

Equation 36

$$\left((1/2 - f_k) - \sqrt{\beta^2 + (1/2 - \delta)^2}\right)^2 + \alpha^2 = f_k^2$$

where, for convenience, the origin has been shifted to α_2 and the positive direction of α has been reversed. In this revised coordinate system, the part of the torus corresponding to the knuckle region of the head is bounded by $0 \le \alpha \le \alpha_2 - \alpha_1$, $-1/2 \le \beta \le 1/2$, and $0 \le \delta \le 1$.

For liquid depths below the lowest point of the dish and above the highest point of the dish, that is when $0 \le \delta \le 1/2 - \beta_1$ and when $1/2 + \beta_1 \le \delta \le 1$, the intersection of a horizontal plane at liquid depth δ with the ring torus of which the knuckle of the left head is a part lies entirely within the knuckle. Figure 5 shows a top view of the knuckle of the left head, rotated clockwise 90 degrees. The curved dashed lines correspond to the inner surface of the knuckle at $\delta = 1/2$, its widest point; the horizontal dashed lines are the planes where the dish joins the knuckle (top line) and where the knuckle joins the flange (bottom line). The solid line is the intersection of the plane $\delta = 1/2 - \beta_1$ with the knuckle. From Figure 5, we see that the cross-sectional area for the knuckle of the left head for any vertical position in the ranges $0 \le \delta \le 1/2 - \beta_1$ and $1/2 + \beta_1 \le \delta \le 1$ is

Equation 37

$$A_2^H = 2D_i^2\int_0^{\beta_f}\alpha d\beta = 2D_i^2\int_0^{\beta_f}\sqrt{f_k^2 - \left((1/2 - f_k) - \sqrt{\beta^2 + (1/2-\delta)^2}\right)^2}\, d\beta$$

wherein β_f is the distance along the line of intersection of the transverse plane where the knuckle joins the flange and the horizontal plane at liquid depth δ, from the centre of the knuckle to its inner surface.

Equation 38

$$\beta_f = \sqrt{1/4 - (1/2 - \delta)^2}$$

For all other liquid depths, that is when $1/2 - \beta_1 < \delta < 1/2 + \beta_1$, the intersection of a horizontal plane at liquid depth δ with the torus of which the knuckle of the left head is a part extends outside of the knuckle as shown in Figure 6. In this instance, the solid black line is the intersection of the plane $\delta = (1/2 - \beta_1 + 2f_k)/2$ with the ring torus of which the knuckle is a part. From Figure 6, we see that the cross-sectional area for the knuckle of the left head for any liquid depth in the range $1/2 - \beta_1 < \delta < 1/2 + \beta_1$ is comprised of three parts: two outer roughly triangular parts and a rectangular part. Summing over the parts yields

Equation 39

$$A_2^H = 2D_i^2 \int_{\beta_d}^{\beta_f} \sqrt{f_k^2 - \left((1/2 - f_k) - \sqrt{\beta^2 + (1/2-\delta)^2}\right)^2}\, d\beta + 2D_i^2 \beta_d (\alpha_2 - \alpha_1)$$

wherein β_d is the distance along the line of intersection of the transverse plane where the dish joins the knuckle and the horizontal plane at liquid depth δ, from the centre of the knuckle to its inner surface.

Equation 40

$$\beta_d = \sqrt{\left((1/2 - f_k) + \sqrt{f_k^2 - (\alpha_2 - \alpha_1)^2}\right)^2 - (1/2 - \delta)^2}$$

Summarizing, the cross-sectional area for Region 2 is

Equation 41

$$A_2^H = \begin{cases} 2D_i^2 \int_0^{\beta_f} \sqrt{f_k^2 - \left((1/2 - f_k) - \sqrt{\beta^2 + (1/2-\delta)^2}\right)^2}\, d\beta & \begin{matrix} 0 \le \delta \le 1/2 - \beta_1 \\ 1/2 + \beta_1 \le \delta \le 1 \end{matrix} \\ 2D_i^2 \int_{\beta_d}^{\beta_f} \sqrt{f_k^2 - \left((1/2 - f_k) - \sqrt{\beta^2 + (1/2-\delta)^2}\right)^2}\, d\beta + 2D_i^2 \beta_d (\alpha_2 - \alpha_1) & 1/2 - \beta_1 < \delta < 1/2 + \beta_1 \end{cases}$$

Unfortunately, there is no closed form solution for the integrals in Equation 41. Consequently, the integrations must be performed numerically.

Region 1 – Dish of the Left Head

Looking down from the top of a horizontally oriented vessel, the cross-section of the dish at any liquid depth that lies within the dish, that is when $1/2 - \beta_1 < \delta < 1/2 + \beta_1$, is a segment of a circle. Figure 7 shows the side view and top view of the left end of a horizontally oriented vessel. The top view shows the circular segment for which the area is required.

From the side view, we can see that the dimensionless radius of this circular segment is half of the chord at liquid depth δ of the circle of radius f_d and centre $(0, f_d)$. Applying the mensuration formulae for a circle we have

Equation 42

$$\rho = \sqrt{f_d^2 - (1/2 - \delta)^2}$$

The distance from the centre of the circle to the centre of the chord is $(f_d - \alpha_1)$ so that, again, applying the mensuration formulae for a circle, the cross-sectional area for Region 1 is

Equation 43

$$A_1^H = \begin{cases} 0 & 0 \le \delta \le 1/2 - \beta_1 \\ & 1/2 - \beta_1 \le \delta \le 1 \\ D_i^2 \left(\rho^2 \cos^{-1}\left(\frac{f_d - \alpha_1}{\rho}\right) - (f_d - \alpha_1)\sqrt{\rho^2 - (f_d - \alpha_1)^2} \right) & 1/2 - \beta_1 < \delta < 1/2 + \beta_1 \end{cases}$$

Cross-Sectional Area of the Vessel

Due to the symmetry of the vessel, $A_4^H = A_2^H$ and $A_1^5 = A_1^H$ so that the cross-sectional area of the vessel at any vertical position within the vessel is

Equation 44

$$A^H = 2(A_1^H + A_2^H) + A_3^H$$

Surface Area as a Function of Liquid Depth for Horizontal Vessels

Region 3 – Cylindrical Part of the Vessel

The total surface area of the cylindrical part of the vessel is found by substituting $\alpha = \alpha_3$ into Equation 29:

Equation 45

$$S_3 = \pi D_i^2(\alpha_3 - \alpha_2)$$

Since the transverse cross-section for the cylindrical part of the vessel is unchanging, the proportion of its surface area below a liquid depth is simply the proportion of the circumference of its transverse cross-section that is below the liquid depth, as shown in Figure 4. Applying the mensuration formulae for a circle, the liquid surface area in Region 3 is

Equation 46

$$s_3^H = S_3 \frac{\theta}{2\pi} = D_i^2(\alpha_3 - \alpha_2)\cos^{-1}(1 - 2\delta)$$

Region 2 – Knuckle of the Left Head

Obtaining the formula for the liquid surface area for the knuckle of the left head is more complicated than for the cylindrical part of the vessel because the radius of the transverse cross-section within the knuckle changes with axial position. Consequently, the surface area must be obtained by integrating the surface area below the liquid depth of infinitesimally thin slices over the knuckle.

For vertically oriented vessels, Equation 27 gives the surface area of a truncated conical slice of the knuckle of the bottom head. For a horizontally oriented vessel, this incremental surface area must be multiplied by the proportion of its circumference that is below the liquid depth.

Figure 8 shows a side view and end view of the left end of a horizontally oriented vessel. The inner most circle shown in the end view is the transverse cross-section of the vessel where the dish joins the knuckle of the head. The outer most circle is the transverse cross-section of the vessel where the knuckle joins the flange of the head, that is, the transverse cross-section of the cylindrical part of the vessel. The dashed circle is the cross-section of the integration slice within the knuckle. This cross-section has radius β. Hence, that the proportion of the circumference of the cross-section that is below the vertical position is

Equation 47

$$\frac{\theta}{2\pi} = \frac{\cos^{-1}\left(\frac{1/2 - \min(\delta, 1/2 + \beta)}{\beta}\right)}{\pi}$$

Then, from Equation 27, the surface area of a truncated conical slice of the knuckle of the left head below liquid depth δ is

Equation 48

$$s = 2D_i^2 f_k \frac{\beta \cos^{-1}\left(\frac{1/2 - \min(\delta, 1/2 + \beta)}{\beta}\right)}{\sqrt{f_k^2 - (\alpha - \alpha_2)^2}}$$

Integrating along the axis of the vessel yields

Equation 49

$$s_2^H = \int_{\alpha^T}^{\alpha_2} s(\alpha, \beta(\alpha), \delta) d\alpha$$

where α^T, the lower limit of the integration, depends on the liquid depth.

When the liquid depth is below the lowest point of the dish, that is when $\delta < 1/2 - \beta_1$, α^T is the axial position corresponding to the maximum distance that a horizontal plane at vertical position δ reaches into the knuckle. For $\delta < 1/2 - \beta_1$ we have $\beta = 1/2 - \delta$. Substituting into Equation 8 and solving for α yields

Equation 50

$$\alpha^T = \alpha_k(\delta) = \alpha_2 - \sqrt{f_k^2 - (f_k - \delta)^2}$$

When the liquid depth is within the dish, that is when $1/2 - \beta_1 \leq \delta \leq 1/2 + \beta_1$, the maximum distance that a horizontal plane at liquid depth δ reaches into the head passes through the knuckle and into the dish so that $\alpha^T = \alpha_1$.

When the liquid depth is above the highest point in the dish, that is when $\delta > 1/2 + \beta_1$, part or the entire knuckle lies completely below the liquid depth. Better numerical accuracy is obtained by segmenting the integration interval into two parts: the left part of the knuckle which is completely below the liquid depth and the right part making up the reminder of the knuckle. The axial position demarking the two parts is the axial position corresponding to the maximum distance that a horizontal plane at the liquid depth reaches into the knuckle, which for $\delta > 1/2 + \beta_1$ is $\alpha_k(1 - \delta)$. For the left part of the knuckle, the surface area is simply given by $s_2^V(\alpha)$ from Equation 28 evaluated at $\alpha = \alpha_k(1 - \delta)$. Hence for $\delta > 1/2 + \beta_1$, we have

Equation 51

$$s_2^H = s_2^V\big(\alpha_k(1 - \delta)\big) + \int_{\alpha_k(1-\delta)}^{\alpha_2} s(\alpha, \beta(\alpha), \delta) d\alpha$$

Summarizing, the liquid surface area for Region 2 is

Equation 52

$$s_2^H = \begin{cases} \int_{\alpha_k(\delta)}^{\alpha_2} s(\alpha, \beta(\alpha), \delta) d\alpha & \delta < 1/2 - \beta_1 \\ \int_{\alpha_1}^{\alpha_2} s(\alpha, \beta(\alpha), \delta) d\alpha & 1/2 - \beta_1 \leq \delta \leq 1/2 + \beta_1 \\ s_2^V\big(\alpha_k(1 - \delta)\big) + \int_{\alpha_k(1-\delta)}^{\alpha_2} s(\alpha, \beta(\alpha), \delta) d\alpha & \delta > 1/2 + \beta_1 \end{cases}$$

where $s(\alpha, \beta(\alpha), \delta)$ is defined by Equation 48 and $\beta(\alpha)$ is solved from Equation 8. As there is no closed form solution for the integrals in Equation 52, the integration must be performed numerically.

Region 1 – Dish of the Left Head

The formula for liquid surface area of the dish for the left head is obtained completely analogously to the corresponding formula for the knuckle. The resulting formulae are

Equation 53

$$s_1^H = \begin{cases} 0 & \delta \le 1/2 - \beta_1 \\ \int_{\alpha_d(\delta)}^{\alpha_1} s(\beta(\alpha), \delta) d\alpha & 1/2 - \beta_1 < \delta < 1/2 \\ s_1^V\big(\alpha_d(1-\delta)\big) + \int_{\alpha_d(1-\delta)}^{\alpha_1} s(\beta(\alpha), \delta) d\alpha & 1/2 \le \delta < 1/2 + \beta_1 \\ S_1 & \delta \ge 1/2 + \beta_1 \end{cases}$$

where

Equation 54

$$s = 2D_i^2 f_d \cos^{-1}\left(\frac{1/2 - \delta}{\beta}\right) d\alpha$$

and

Equation 55

$$\alpha_d(\delta) = f_d - \sqrt{f_d^2 - (1/2 - \delta)^2}$$

Again, as there is no closed form solution for the integral in Equation 53, the integration must be performed numerically.

Surface Area of the Vessel

Due to symmetry of the vessel, $s_4^H = s_2^H$ and $s_5^H = s_1^H$. Hence, the liquid surface area for the vessel is

Equation 56

$$s = 2(s_1^H + s_2^H) + s_3^H$$

Results

The formulae in this paper have been implemented in a Microsoft Excel workbook "Vessel Geometry Calculations" which is available on Honeywell's UniSim® Design web page, on the Documentation tab. Configuration of the vessel is performed through the Vessel Configuration and Head Styles tables, depicted in Figure 9 and Figure 10. The head style, inside diameter, either tan-tan height/length or inside height/length, and wall thickness of the vessel must be specified. Quantities to be specified are highlighted in a burnt orange color. The Head Style table lists the available head styles. For the User Specified head style, a descriptor for the head style, the dish radius factor, the knuckle radius factor or the knuckle radius, and the reference diameter must be specified. Comments embedded in the worksheet provide guidance and conditional formatting highlights invalid entries in red.

The values for quantities calculated for the vessel are shown in three tables, depicted in Figure 11, Figure 12 and Figure 13. The Head Style Parameters table shows the key dimensionless axial positions and radii, and the dish and knuckle radius factors. The height/length, surface area and capacity calculated for each of the five regions of the vessel are shown in the Vessel Regions table. The Calculator table shows the liquid depth, cross-sectional area, liquid surface area and liquid volume for the specified percent liquid depth. Finally, the Generate Tables and Plots button initiates the generation of tables and plots for these quantities over the full range of percent liquid depth.

The Excel workbook is set up with the example used by Crookston and Crookston: a vessel with ASME F&D heads, internal diameter of 100 inches, and inside height/length of 120 inches, oriented vertically and horizontally. The only difference is that the outside diameter is used as the reference diameter for the ASME F&D head type rather than the inside diameter as per Crookston and Crookston.

Figure 14 and Figure 15 are plots of the dimensionless liquid surface area and dimensionless cross-sectional area for this example with the vessel oriented both vertically and horizontally. Simpson's Rule with 1000 intervals is used to evaluate the integrals associated with the horizontal orientation. Figure 14 shows that the liquid surface area is not strongly dependent on the orientation of the vessel. Consequently, heat transfer between the fluid holdup in the vessel and the inner surface of the vessel walls will not be strongly dependent on the orientation of the vessel. However, Figure 15 shows that cross-sectional area is strongly dependent on the orientation of the vessel so that heat transfer and mass transfer between the zones of a multi-zone vessel model, as is required when modeling emergency depressuring, will be strongly influenced by the orientation of the vessel.

Correctness of the formulae for liquid surface area can be checked by comparing the calculated values for liquid depths of 50% and 100% filled for the same vessel oriented vertically and horizontally. The liquid surface areas for the combined heads, the cylinder of the vessel, and the overall vessel should be the same for these two liquid depths. This comparison has been made for the example above. Table 2 and Table 3 show that the dimensionless liquid surface areas are the same to six and seven significant digits, giving confidence in the correctness of the formulae.

The liquid surface area and the cross-sectional area of the heads can also be checked by comparing their calculated values for the same spherical vessel oriented vertically and horizontally. In this instance, the surface area and the cross-sectional area should be the same for all liquid depths. A spherical vessel was straightforwardly constructed in the Excel spreadsheet by selecting the hemispherical head style and setting the tan-tan-length of the vessel to zero. Table 4 shows good agreement between the calculated values. At worst, the corresponding surface areas are the same to three and a half significant digits. The corresponding cross-sectional areas are the same to five significant digits. These results give further confidence in the correctness of the formulae.

In fact, the accuracy of the numerical integrations performed for horizontally oriented vessels are least accurate for spherical vessels. The number of integration intervals used for the Simpson's Rule integrations is the same for all head styles. Since the range of the integrations is the largest for the hemispherical head style, the integration interval is correspondingly largest and, hence, the integration accuracy poorest. This discrepancy could be remedied by making the number of integration intervals dependent on the size of the integration range.

Earlier we noted that rigorous treatment of the liquid volume, liquid surface area and cross-sectional area of vessels with dished heads is particularly important when simulating critical process safety operations. In fact, this was our principal motivation for extending the work of Crookston and Crookston to liquid surface area and cross-sectional area. The formulae presented herein for liquid surface area and cross-sectional area and the formulae presented by Crookston and Crookston for liquid volume are incorporated in the multi-zone vessel model used in the Blowdown Utility of the UniSim Design process engineering simulator. Because blowdown of vessels is a critical safety operation, accurate prediction of the process conditions within the vessel and the temperatures within the vessel walls during depressuring is a critical element in creating a process design that will ensure safe operation during blowdown with least capital cost. The fluid holdup in the vessel is represented by three equilibrium zones nominally corresponding to the vapour, liquid and aqueous holdup in the vessel. Each zone incorporates heat transfer with the vessel walls, heat transfer and mass exchange with adjacent zones, and heat transfer with the environment through heat conduction in the vessel walls and encasing insulation. The use of rigorous formulae for liquid volume, liquid surface area and cross-sectional area, together with highly accurate numerical integration when required, contribute to the accuracy of the model and, hence, confidence in its predictions.

A depressuring simulation for the vessel used in Crookston and Crookston's example was performed in UniSim Design. The vessel is vertically oriented, constructed of carbon steel with thickness of 60 mm, and is not insulated. Depressuring occurs through a relief orifice with a diameter of 15 mm connected to a nozzle located at the top of the vessel. For the simulation, the vessel was initialized with a three phase mixture of methane, hexane and water at about 10 C and 3000 kPa such that the vessel was initially filled with about 20% liquid. Figure 16 is a screenshot of the results for the depressuring simulation. The strip chart for the pressure (upper right corner) shows that depressuring is more or less complete after 20 minutes. The other strip charts show the inside and outside wall temperatures for the vapour, hydrocarbon liquid and aqueous zones in the vessel. In this simulation, the lowest temperature is encountered at the inside surface of the wall in the hydrocarbon liquid zone.

Notation

A	cross-sectional area
c	length of a chord of a circle
D_i	inside diameter of the flange part of the head and the cylinder of the vessel
D_o	outside diameter of the flange part of the head and the cylinder of the vessel
f_d	dish radius factor
f_d^F	dish radius factor as provided by fabricator or standard
f_k	knuckle radius factor
f_k^F	knuckle radius factor as provided by fabricator or standard
H	total inside height or length of the vessel
H_f	flange height or length
H_{tt}	tan-tan height or length of the vessel
R_d	inside radius of the dish part of the head
R_k	inside radius of the knuckle part of the head
s	liquid surface area, surface area
S	total surface area
t	thickness of the head and the cylinder of the vessel
v	volume
V	capacity
x	dimensional radial position, dimensional vessel radius
y	dimensional axial position
α	dimensionless axial position
α_d	axial position corresponding to the maximum distance a horizontal plane reaches into the dish of the left head of a horizontally oriented vessel
α_k	axial position corresponding to the maximum distance a horizontal plane reaches into the knuckle of the left head of a horizontally oriented vessel
α^T	axial position lower integration limit
β	dimensionless radial position, dimensionless vessel radius
β_d	distance along the line of intersection of the transverse plane where the dish joins the knuckle and a horizontal plane within the dish of a horizontally oriented vessel, from the centre of the knuckle to its inner surface.

β_f distance along the line of intersection of the transverse plane where the knuckle joins the flange and a horizontal plane within the knuckle of a horizontally oriented vessel, from the centre of the knuckle to its inner surface.

δ dimensionless liquid depth in a horizontally oriented vessel

θ angle of a circular sector formed by the intersection of a transverse plane with a horizontally oriented vessel with liquid depth δ

ρ radius of the circle formed by the intersection of a horizontal plane with the dish part of the head of a horizontally oriented vessel

σ dimensionless length of the side of a truncated cone integration slice

Superscripts

H horizontally oriented vessel

V vertically oriented vessel

Numeric Subscripts

1. transverse plane where the dish joins the knuckle of the bottom or left head; dish of the bottom or left head
2. transverse plane where the knuckle joins the flange of the bottom or left head; knuckle of the bottom or left head
3. transverse plane where the flange joins the knuckle of the top or right head; cylindrical part of the vessel
4. transverse plane where the knuckle joins the dish of the top or right head; knuckle of the top or right head
5. transverse plane at the top or right end of the vessel; dish of the top or right head

References

1. Crookston, D.R. and Crookston, R.B., “Calculate Liquid Volumes in Tanks with Dished Heads,” *Chemical Engineering*, September, pp. 55-63, 2011.
2. Selby, S.M., “Standard Mathematical Tables,” 19th Edition, The Chemical Rubber Co., 1971.

Figures

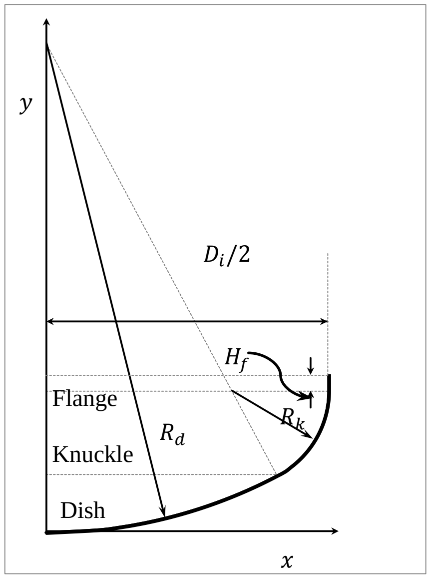

Figure 1. Cross-sectional view of a torispherical head showing the radii of curvature and the coordinate system used for vertically oriented vessels

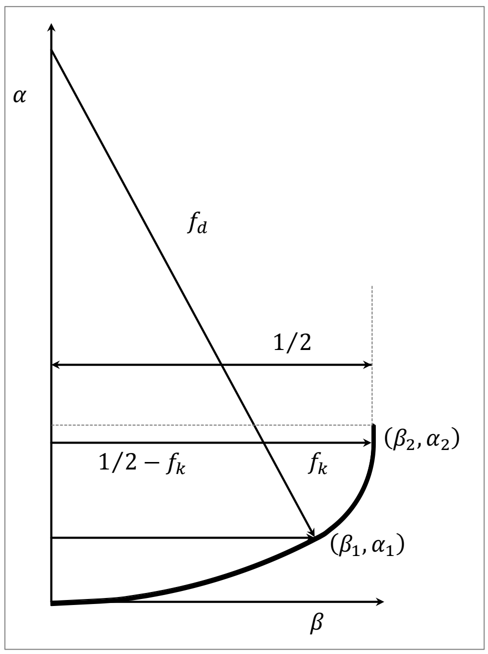

Figure 2. Cross-sectional view of a torispherical head showing dimensionless parameters

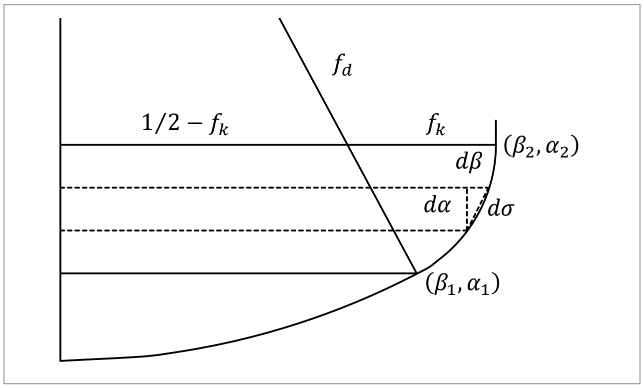

Figure 3. Thin horizontal slice of the knuckle for surface area integral

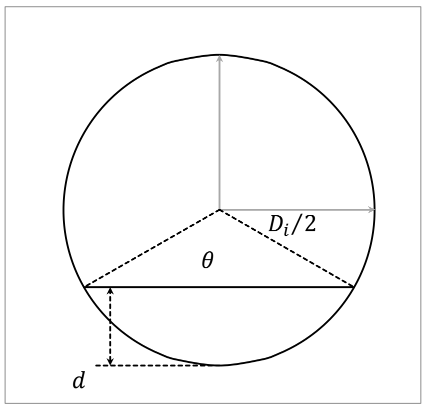

Figure 4. End view of the intersection of a horizontal plane with the cylindrical part of a horizontally oriented vessel

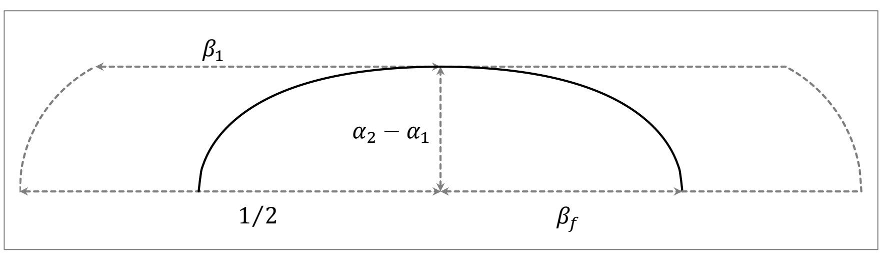

Figure 5. Top view of the knuckle of the left head of a horizontally oriented vessel, rotated 90 degrees clockwise, showing the intersection of the horizontal plane at vertical position $\boldsymbol{\delta} = \mathbf{1/2} - \boldsymbol{\beta_1}$ with the knuckle

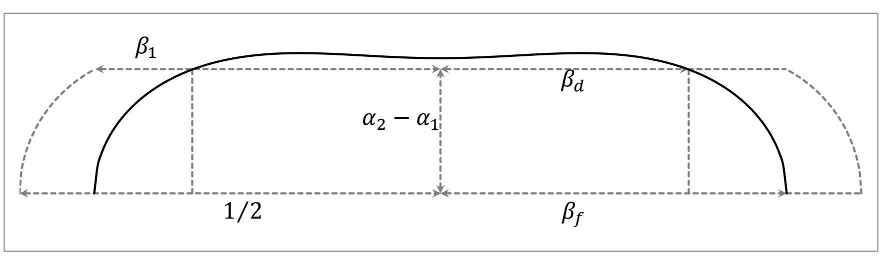

Figure 6. Top view of the knuckle of the left head of a horizontally oriented vessel, rotated 90 degrees clockwise, showing the intersection of the horizontal plane at vertical position $\boldsymbol{\delta} = (\mathbf{1/2} - \boldsymbol{\beta_1} + \mathbf{2}\boldsymbol{f_k})/\mathbf{2}$ with the knuckle

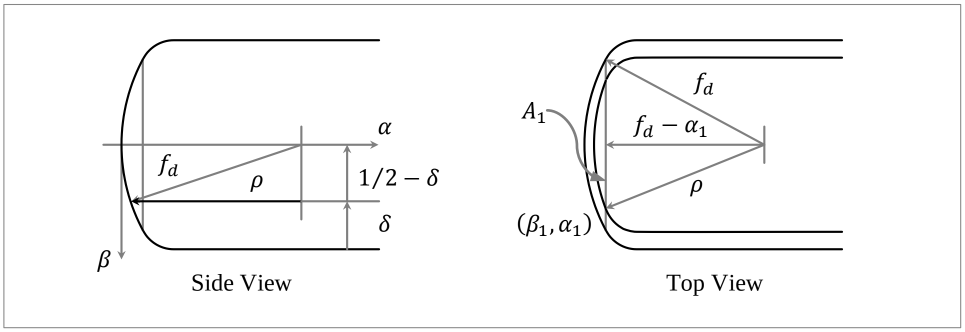

Figure 7. Side view and top view of the left end of a horizontally oriented vessel showing liquid depth

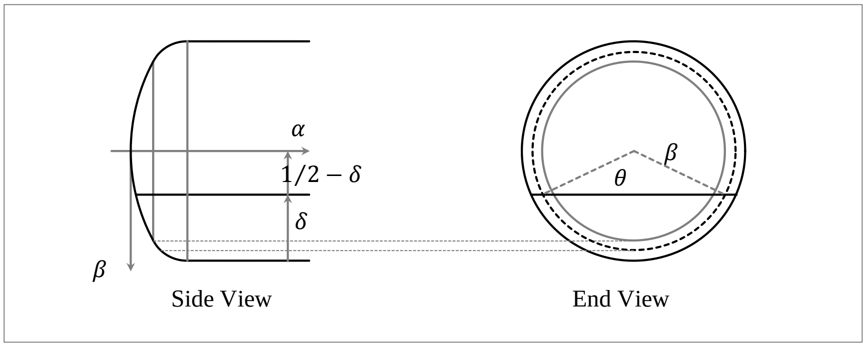

Figure 8. Side view and end view of the left end of a horizontally oriented vessel showing liquid depth $\boldsymbol{\delta}$

Vessel Configuration	
Vessel Name	V-101
Head Style Number	3
Inside Diameter	2.54
Height/Length Specification	
Tan-Tan Height/Length (m)	
Inside Height/Length (m)	3.048
Wall Thickness (m)	0.05

Figure 9. Vessel Configuration table of the Excel workbook

Head Style		f_d^F	f_k^F	D_R	R_k (mm)
1	Hemispherical	0.500	0.500	Inside	-
2	2:1 Semi-Elliptical	0.900	0.170	Inside	-
3	ASME F&D	1.000	0.060	Outside	-
4	ASME 80/10 F&D	0.800	0.100	Outside	-
5	ASME 80/6 F&D	0.800	0.060	Outside	-
6	Standard F&D	1.000	-	Outside	50
7	Shallow F&D	1.500	-	Outside	50
8	DIN 28011	1.000	0.100	Outside	-
9	DIN 28013	0.800	0.154	Outside	-
10	User Specified	0.900	0.170	Inside	

Figure 10. Head Styles table of the Excel workbook

Head Style Parameters			
α_5	1.2	β_5	0.0
α_4	1.089894555	β_4	0.46557212
α_3	1.034138677	β_3	0.5
α_2	0.165861323	β_2	0.5
α_1	0.110105445	β_1	0.46557212
f_d	1.039370079	$1/2 - \beta_1$	0.03442788
f_k	0.062362205	$1/2 + \beta_1$	0.96557212

Figure 11. Head Style Parameters table of the Excel workbook

Vessel Regions			
Region	Height/Length (m)	Surface Area (m^2)	Capacity (m^3)
5	3.048	4.639	0.626
4	2.768	1.365	0.690
3	2.627	17.599	11.175
2	0.421	1.365	0.690
1	0.280	4.639	0.626
Total	-	29.606	13.807

Figure 12. Vessel Regions table of the Excel workbook

Calculator	Vertical	Horizontal
Liquid Depth (%)	50	50
α	0.600	-
Liquid Depth (m)	1.524	1.270
Cross-sectional Area (m^2)	5.067	7.199
Liquid Surface Area (m^2)	14.803	14.803
Liquid Volume (m^3)	6.903	6.903

Generate Tables and Plots

Figure 13. Calculator table of the Excel workbook

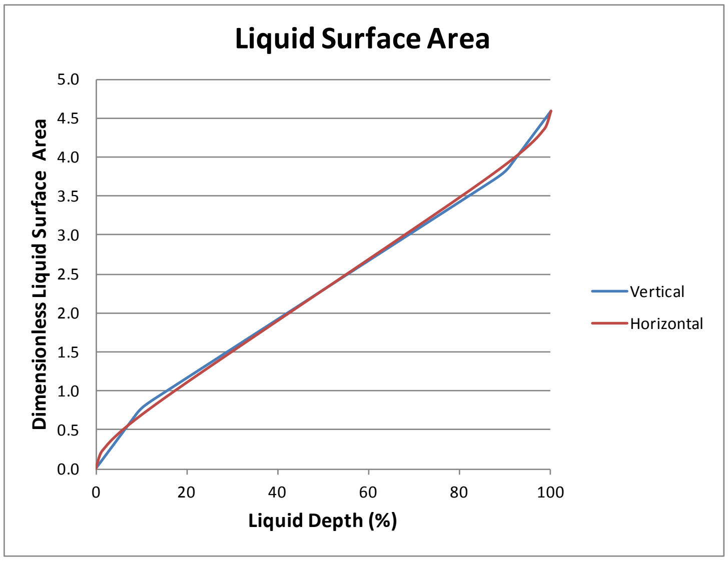

Figure 14. Liquid surface area versus liquid depth for the same vessel oriented vertically and horizontally

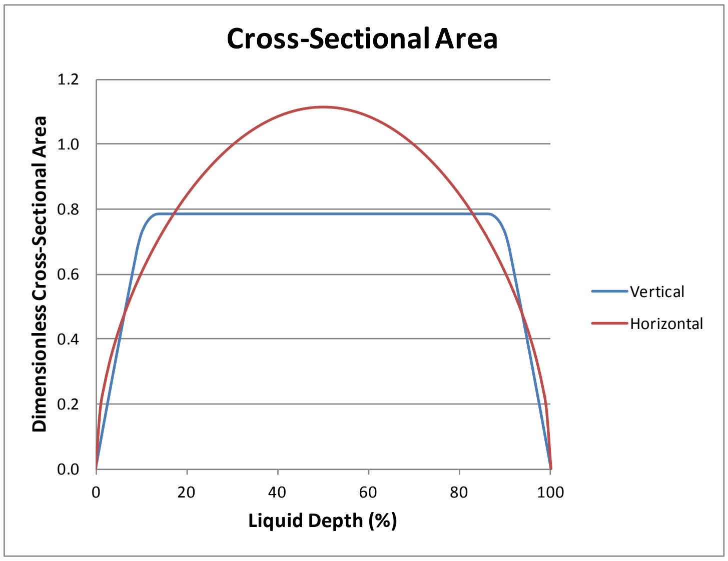

Figure 15. Cross-sectional area versus liquid depth for the same vessel oriented vertically and horizontally

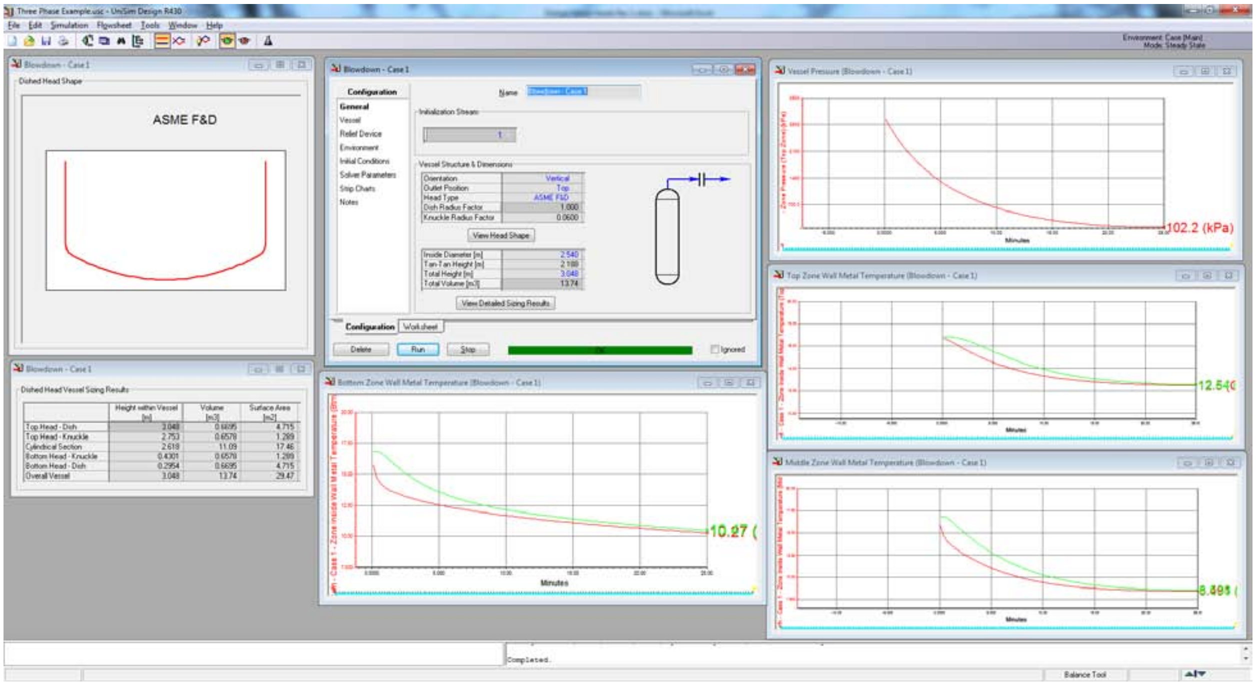

Figure 16. UniSim Design Blowdown Utility screenshot – three phase depressuring

Tables

Table 1. Standard and manufacturer-specific torispherical head styles

Torispherical Head Style	f_d^F	f_k^F	Reference Diameter, D_R, for f_d^F and f_k^F
ASME F&D [5]	1.0	0.060	D_o
ASME 80/10 F&D [5]	0.8	0.100	D_o
ASME 80/6 F&D [5]	0.8	0.060	D_o
1.9:1 Semi-Elliptical [1,2]	1/1.16	1/5.39	D_i
2:1 Semi-Elliptical [1,2,3] [5]	 0.9 0.8	 0.170 0.150	 D_i D_o
Hemispherical	0.5	0.500	D_i
Standard F&D [1,3] [5]	 1.0 1.0	 15 – 50 mm $/D_R$ $3t_k/D_R$	 D_o D_o
Shallow F&D [1,3] [4] [5]	 1.3 1.5 – 2.0 1.5	 15 – 50 mm $/D_R$ 32 mm, 51 mm, 76 mm $/D_R$ $3t_k/D_R$	 D_o D_i D_o
DIN 28011 [1,2,3]	1.0	0.100	D_o
DIN 28013 [1,2,3,4]	0.8	0.154	D_o

1. König & Co GmbH, http://www.koenig-co.de/
2. Slawinski & Co. GmbH, http://www.slawinski.de/en/history/
3. Weisstaler Bödenpresswerke GmbH, http://www.weisstaler.de/
4. Dished Heads, http://dishedheads.com.au/
5. Brighton Tru-Edge Heads, http://www.brightontruedge.com/

Table 2. Dimensionless liquid surface area at 50% liquid depth for the same vessel oriented vertically and horizontally

Vertical		Horizontal	
Dish – Bottom	0.719050	Dish – Left	0.359525
Knuckle – Bottom	0.211563	Knuckle – Left	0.105782
Head – Bottom	0.930613	Head – Left	0.465307
Cylinder	1.363887	Cylinder	1.363887
Knuckle – Top	0.0	Knuckle – Right	0.105782
Dish – Top	0.0	Dish – Right	0.359525
Head – Top	0.0	Head – Right	0.465307
Heads	0.930613	Heads	0.930613
Vessel	2.294500	Vessel	2.294500

Table 3. Dimensionless liquid surface area at 100% liquid depth for the same vessel oriented vertically and horizontally

Vertical		Horizontal	
Dish – Bottom	0.719050	Dish – Left	0.719050
Knuckle – Bottom	0.211563	Knuckle – Left	0.211563
Head – Bottom	0.930613	Head – Left	0.930613
Cylinder	2.727774	Cylinder	2.727774
Knuckle – Top	0.211563	Knuckle – Right	0.211563
Dish – Top	0.719050	Dish – Right	0.719050
Head – Top	0.930613	Head – Right	0.930613
Heads	1.861226	Heads	1.861226
Vessel	4.589000	Vessel	4.589000

Table 4. Dimensionless liquid surface area and dimensional cross-sectional area the same spherical vessel oriented vertically and horizontally

Liquid Depth (%)	Dimensionless Liquid Surface Area		Dimensionless Cross-sectional Area	
	Vertical		Horizontal	
0	0.00000	0.00000	0.00000	0.00000
10	0.31416	0.31416	0.28274	0.28274
20	0.62832	0.62832	0.50266	0.50266
30	0.94248	0.94248	0.65973	0.65973
40	1.25664	1.25663	0.75398	0.75398
50	1.57080	1.57080	0.78540	0.78540
60	1.88496	1.88446	0.75398	0.75398
70	2.19912	2.19868	0.65973	0.65973
80	2.51327	2.51296	0.50266	0.50266
90	2.82743	2.82733	0.28274	0.28274
100	3.14159	3.14159	0.00000	0.00000

For More Information
Learn more about how Honeywell's Process Design can help to calculate areas, visit our website www.honeywellprocess.com/software or contact your Honeywell account manager.

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WP-14-03-ENG
January 2014

Honeywell